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A kinetic approach to some quasi-linear laws of macroeconomics

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Abstract. Some previous works have presented the data on wealth and income distributions in developed countries and have found that the great majority of population is described by an exponential distribution, which results in idea that the kinetic approach could be adequate to describe this empirical evidence. The aim of our paper is to extend this framework by developing a systematic kinetic approach of the socio-economic systems and to explain how linear laws, modelling correlations between macroeconomic variables, may arise in this context. Firstly we construct the Boltzmann kinetic equation for an idealised system composed by many individuals (workers, officers, business men, etc.), each of them getting a certain income and spending money for their needs. To each individual a certain time variable amount of money is associated – this meaning him/her phase space coordinate. In this way the exponential distribution of money in a closed economy is explicitly found. The extension of this result, including states near the equilibrium, give us the possibility to take into account the regular increase of the total amount of money, according to the modern economic theories. The Kubo-Green-Onsager linear response theory leads us to a set of linear equations between some macroeconomic variables. Finally, the validity of such laws is discussed in relation with the time reversal symmetry and is tested empirically using some macroeconomic time series.

PACS. 87.23.Ge Dynamics of social systems – 02.50.-r Probability theory, stochastic processes, and statistics – 89.90.+n Other topics of general interest to physicists

1 Introduction

The use of concepts from statistical physics in the description of financial, economic and financial systems has already an ample history, defining the new fields of research called econophysics [1] and sociophysics [2]. Indeed, during the last decade, hundreds of papers and books have been published to gain new insights into problems traditionally not associated with physics; at the same time, concepts such as "scale invariance", "critical point" or "steady states" have already enriched the vocabulary of many economists and sociologists. A lot of models taken over physics emphasize one or more features of the complex economic phenomena without pretension to an exhaustive picture. Note that the recent papers operate a non-ambiguous distinction between the microeconomic-financial and macroeconomic-social levels of description, as well as between the microscopic (statistical) and macroscopic (phenomenological) physics. For example, the stock market crashes cannot be treated in the same way as the global inflation; the statistics of price fluctuations has little to do with the statistics of unemployment, etc.

Our paper focuses on the macroeconomic level of description of a socio-economic system. At this level money plays an essential role, like the blood into a body. It is not surprisingly that all classical (and topical) economic theories allocate large spaces to the movement of money (see *e.g.* [3]). The recent econophysics literature includes also some contributions on this question: Donangelo and Sneppen [4] modelled the evolution of money from unsuccessful barter attempts; recently, this model was modified by a deterministic instead of a probabilistic selection of the most desired product as money [5]; Bak *et al.* [6] proposed a dynamical many-body theory of money in which the value of money is a "strategic variable" that is chosen by the individual agents. The models cited above have the support of some interesting numerical simulations but their results have been less compared with the empirical (statistical) data. Also, due to their simplicity of principle, these models do not attain to the fundamentals of statistical physics. A genuine kinetic approach of an economic system is that proposed by Ausloos [7], which includes the main features of a gas-kinetic theory: a phase space density, a Boltzmann kinetic equation which results in some conservation equations; there is also a pressure and a temperature (seen as the inverse of a relaxation time that is considered the same for all agents). Since the approach is

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restricted to the stock market prices, it does not lead to results in agreement with the empirical data. In particular, the Gaussian distribution of price fluctuations is not confirmed empirically. It appears that the price movement in the stock market space has little to do with the Boltzmann's stosszahlansatz (molecular chaos), which cannot explain some collective effects (*e.g.* the "herd" effect, the influences between traders, etc.).

On the other hand, an empirical study by Montroll [8] showed that, at macroeconomic scale, the prices of various items follow a Gaussian distribution with a variance that is approximately independent to the inflation rate (the data taken over from annual Sears, Roebuck and Co. catalogues refer to 1916, 1924–25 and 1974–75). Moreover, the author found that for the great majority of population the incomes follow also a Gaussian distribution, and explained these results by the maximization of an entropy function for socio-technical systems. The recent works of Dragulescu and Yakovenko [9,10] constitutes important steps forward on this way, establishing the relation between the Boltzmann kinetic equation and the conservation of money in a closed economic system. By introducing an effective temperature (equal to the average amount of money per economic agent) they found the exponential Boltzmann-Gibbs distribution of money (and of income, for about 95% of population), in excellent agreement both with the results of computer simulation and the fitted empirical data [10].

In the framework of modern economic theories, which restored the role of money in economic dynamics, we could ask if the total amount of money M, is indeed conserved, at least at short-time scales. The answer seems to be negative since the regular increase of the quantity of money tends to the status of an economic law [11]. In fact, M can be seen as an extensive variable like the energy in physical systems, but its conservation appears as an idealisation similar to the "isolated system" from physics. Somehow similar situation we meet when we define the concept of efficient market. A market is said to be efficient if all the available information is instantly processed when it reaches the market and it is immediately reflected in a new value of prices of the assets traded. Although the efficient market is an idealised system that only approximates real markets, this concept is useful in any attempt to model the real markets behaviour. In the framework of our model, the increase of M is in relation with other variables intensive or extensive. Some of these variables belong to the macroeconomic level (inflation, rate of interest), other have a strong social character (un-employment, income/money). Our goal is to find such relations in the proximity of the equilibrium states.

As we have already shown, both Gaussian (Maxwell) distributions and Boltzmann-Gibbs (exponential) distributions have been identified in the studies of socio-economic structure. The aim of our paper is to extend this framework by developing a systematic kinetic approach of the socio-economic systems and to explain how linear laws, modelling correlations between macroeconomic variables, may arise in this context. In Section 2 we construct the Boltzmann kinetic equation for an idealised system composed by many individuals (workers, officers, business men, etc.), each of them getting a certain income and spending money for their needs. To each individual a certain time variable amount of money is associated – this meaning him/her phase space coordinate [12]. In this way the exponential distribution of money in a closed economy is explicitly found (Sect. 3). In the same section we study also the variation of Gini coefficient (which measures the inequality of wealth distribution [13]) for some East-European countries during the last years. The extension of these results, including states near the equilibrium, give us the possibility to take into account the regular increase of the total amount of money, according to the modern economic theories. The Kubo-Green-Onsager linear response theory leads us to a set of linear equations between some macroeconomic variables (Sect. 4). Finally, the validity of such laws is discussed in relation with the time reversal symmetry (Sect. 5) and is tested empirically using some macroeconomic time series (Sect. 6). Here we establish the correlations between the economic variables seen as generalized forces or fluxes. The conclusions and some possible extensions are discussed in Section 7.

2 The kinetic level of description

Our system consists in N individuals labeled by the index $j(j = 1 \ldots N)$. According with the model treated in reference [9], we consider as an individual variable the amount of money possessed by each individual (a quantity positive semi-definite): $m_j \geq 0$ for any $j = 1 \ldots N$. In this way we can choose the relevant variables and to construct the phase space for the space number density, ρ. As macroscopic variables could be chosen the total amount of money (M), the natural unemployment (U) *i.e.* the unemployment corresponding to the abstention of inflation [11], or any other quantities that are conserved at equilibrium.

Following the usual methods of statistical physics [12] we can divide the phase space into small cellular volume elements, each volume element having assigned an index $i = 1, 2, 3,$ The variables in the kinetic level of description are the number of individuals $N_i(t)$, which occupy the volume elements. These occupancy numbers are extensive variables since they are proportional to the size of the volume element. They are also stochastic variables since specification of the number of individuals in these volume elements does not provide enough information to specify deterministically which interactions (encounters) will occur. As in reference [7] we consider that the binary encounters dominate so that only two volume elements, located at m and m_1 are involved. At the end of the interaction, each of these volume elements will contain one less individual, while a new person will appear in volume elements located at m' and m'_1. If we consider these volume elements to be infinitesimal, the extensive property of the occupancy numbers can be used to introduce the space number density $\rho(m, t)$. Thus means the number of individuals with coordinates in the range $[m, m + \mathrm{d}m]$. Since encounters are a binary process, to describe their effect it is necessary to know the density of pairs of individuals in space $\rho^{(2)}(m, m_1, t)$, the so-called pair distribution function. Adopting the Boltzmann's assumption of stosszahlansatz we can write the pair distribution function as a product:

$$\rho^{(2)}(m, m_1, t) = \rho(m, t)\rho(m_1, t). \quad (1)$$

Equations of this kind are frequently met in the study of Markov processes. Furthermore, near stable steady states it is usually possible to find a contracted description that will be stationary, Gaussian and Markovian. The great advantage of this assumption is that it permits a description of the average effect of interactions in terms of ρ, without independently introducing $\rho^{(2)}$. Thus, the dynamics of an encounter can be treated in the same fashion as the dynamics of a molecular beam experiment, leading to the kinetic equation:

$$\frac{\partial \rho}{\partial t} = \int \widehat{\sigma}_T \, g[\rho' \rho'_1 - \rho \rho_1] \mathrm{d}m_1 \cdot \quad (2)$$

In equation (2) we neglect the non-dissipative flux in phase space called streaming: even if no collisions occurred in the time $\mathrm{d}t$, all individuals at the point m would move to a new spatial position $(m + \dot{m}\mathrm{d}m)$. We take into account for the r.h.s. of equation (2) only the source term, representing the dissipative effect of encounters. We used the notations: $g = |m_1 - m|$, $\rho = \rho(m, t)$, $\rho' = \rho(m', t)$, $\rho_1 = \rho(m_1, t)$, $\rho'_1 = \rho(m'_1, t)$. In the framework of the classical theory, the linear operator $\widehat{\sigma}_T$ is related to the differential scattering cross section $\sigma(\Omega, g)$, where Ω is the solid angle: $\widehat{\sigma}_T \, [.] = \int \mathrm{d}\Omega \sigma(\Omega, g)[.]$.

Finally, it is easy to prove the H-theorem: defining the H-function by:

$$H = \iint \rho \ln \rho \mathrm{d}m \quad (3)$$

and following the usual way *i.e.* taking the time derivative of H, making some changes of variables and adding the equations, one get:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{4} \iiint \widehat{\sigma}_T \, g \rho' {\rho'}_1 \times \left[\left(1 - \frac{\rho \rho_1}{\rho' {\rho'}_1} \right) \ln \frac{\rho \rho_1}{\rho' {\rho'}_1} \right] \mathrm{d}m \mathrm{d}m_1 \leq 0 \quad (4)$$

with the equality holding for (and only for) $\rho = \rho^0$ which satisfies:

$$\rho^0 \rho_1^0 = {\rho'}^0 {\rho'}_1^0 \cdot \quad (5)$$

Note that the Boltzmann equation is non-linear and because it accounts fully for binary encounters, it is useful for describing processes both near and far from equilibrium.

3 The equilibrium distribution of money

In this section we recover some results obtained already in literature [8–10]. Taking the logarithm in equation (5) it follows that only distribution which satisfy:

$$\ln \rho^0 + \ln \rho_1^0 = \ln {\rho'}^0 + \ln {\rho'}_1^0 \quad (6)$$

correspond to a constant value of H. As the H-theorem proves that H is a monotonically decreasing function of time, we infer that the phase space density which satisfies equation (6) is the asymptotic equilibrium density function. In the absence of an external field, the equilibrium phase space density will be independent of position, and equation (6) can be write symbolically as:

$$\Xi(m) + \Xi(m_1) = \Xi(m') + \Xi(m'_1) \cdot \quad (7)$$

A function that satisfies the equality (7) is called a collisional invariant. According with [9], in a closed economic system (at least at short-time scales) the total amount of money is conserved: money (the fiat) is not allowed to be manufactured by regular economic agents but can only be transferred. This is equivalent to the conservation of energy.

The density functions that satisfy equation (7) have the general form:

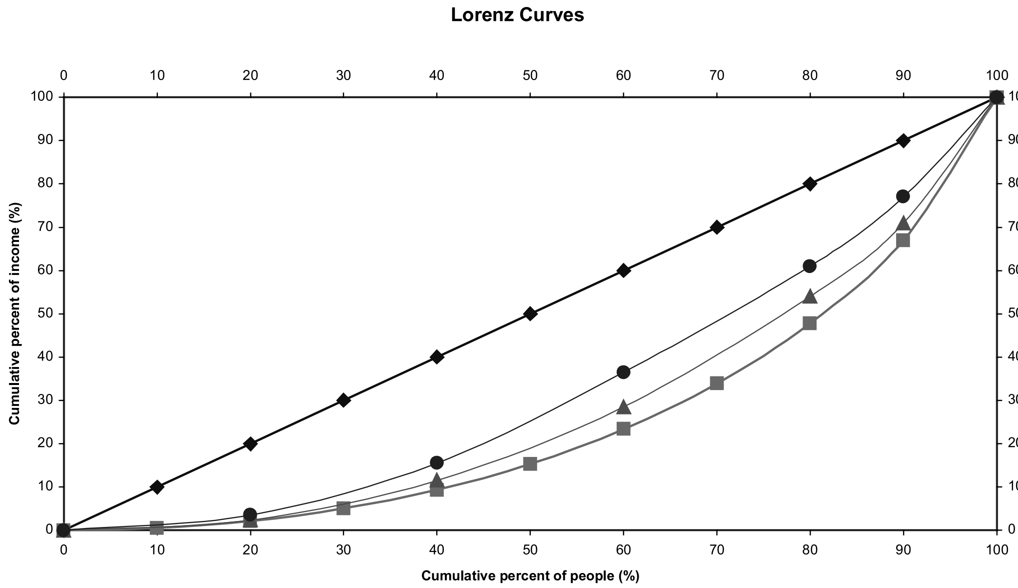
$$\rho^0(m_i) = C \exp\left(-\frac{m_i}{T}\right) \qquad \text{(Boltzmann-Gibbs)} \quad (8)$$

where $\langle\rangle$ denotes a phase space average, and T is an effective temperature equal to the average amount of money per economic agent.

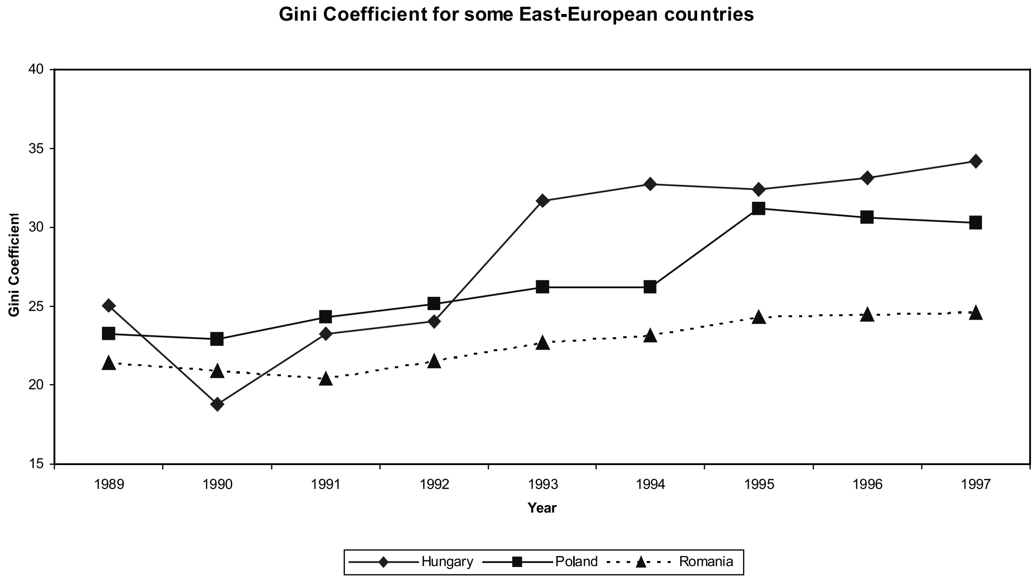
A straightforward graphical interpretation of the Gini coefficient is the Lorenz curve [13], which is the thick curve in Figure 1. The horizontal axis plots the cumulative percentage of the population whose inequality is under consideration, starting from the poorest and ending with the richest. The vertical axis plots the cumulative percentage of income (or expenditure) associated with the units on the horizontal axis.

In the case of a completely egalitarian income distribution in which the whole population has equal incomes, the Lorenz curve would be the dashed straight 45-degree line. When inequality exists, the poor population has a proportionately lower share of income compared with the rich population, and the Lorenz curve may look like the above thick curve below the 45-degree line. As inequality rises, so the thick curve moves towards the bottom right-hand corner. The Gini coefficient is the area A between the 45-degree line and the Lorenz curve divided by 1/2, the total area under the 45-degree line. The Gini coefficient may be given as a proportion or percentage.

An interesting empirical aspect pointed out by Dragulescu and Yakovenko [10] is that for the great majority of population ($\approx 95\%$) the distribution of individual income follows an exponential law. The horizontal axis



(a)



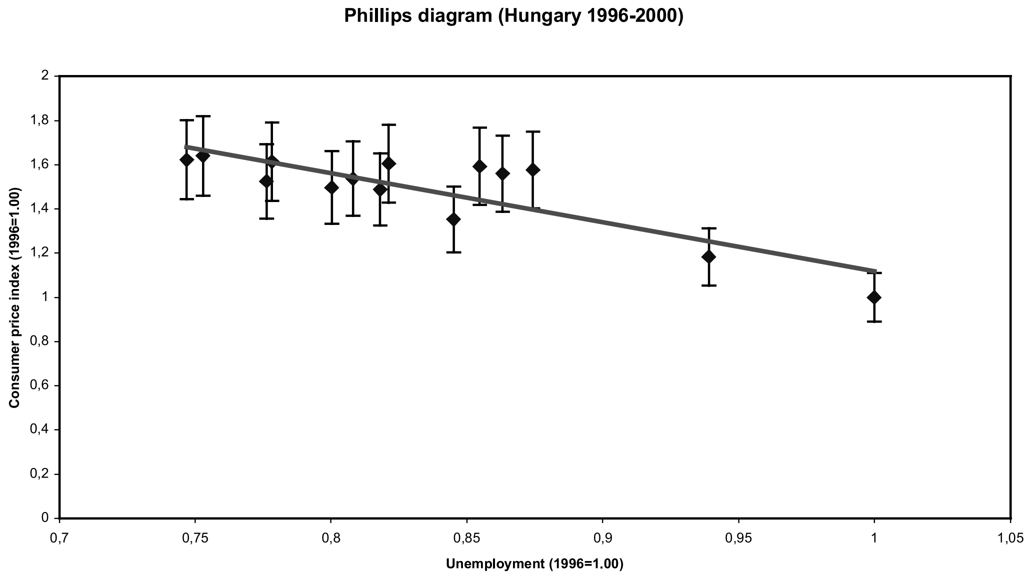
(b)

Fig. 1. (a) Lorenz curves. Solid lines: the straight 45-degree line (Lorenz curve in the case of a completely egalitarian income distribution) and the Lorenz plot in the case of an exponential distribution. The Gini coefficient – the area between the two solid lines divided by the total area under the 45-degree line – is 1/2 (50%). Dashed lines: the Lorenz plots for Romania (1991 – the circles; 1997 – the triangles). (b) The dynamical evolution of Gini coefficient for some East-European countries.

plots the cumulative fraction of the population with income below r, $X(r)$. The vertical axis plots the cumulative fraction of income $Y(r)$ associated with the units on horizontal axis:

$$X(r) = \int_0^r \rho(r')\mathrm{d}r'; \quad Y(r) = \frac{\int_0^r r'\rho(r')\mathrm{d}r'}{\int_0^\infty r'\rho(r')\mathrm{d}r'} \cdot \qquad (9)$$

The Gini coefficient is given by: $G = 2\int_0^1 (X - Y)\mathrm{d}X$. When the distribution is completely egalitarian $G = 0$. If the society's total income accrues to only one person, leaving the rest with no income at all, then $G = 1$ (or 100%). For the exponential distribution $G = 0.5$. This is in good agreement with the values: 0.64–0.68 for United Kingdom and 0.47–0.56 for USA found by Dragulescu and Yakovenko [10]. Note that our analysis refers only to



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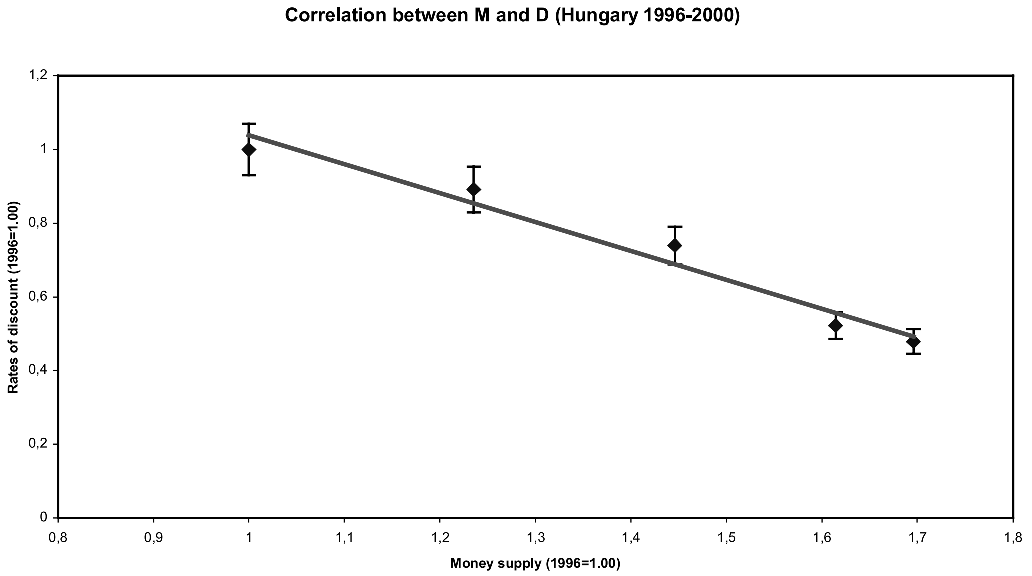


(b)

Fig. 2. (a) Consumer price index (P) *versus* unemployment (U) for Hungary. The empirical data were registered at UNSD database [16] from 1996 to 2000. The values corresponding to 1996 ($P = 382.8\%$ for $P = 100\%$ in 1990; $U = 500.6$ thousands persons aged 16–64) are considered equal to 1.00. Error bars are bootstrap 89% confidence intervals. (b) Rates of discount of central banks (D) *versus* money supply (M) for Hungary. The empirical data were registered at UNSD database [16] from 1996 to 2000. The values corresponding to 1996 ($D = 23.00\%$ per annum; $M = 1237.200$ mill. forints) are considered equal to 1.00. Error bars are bootstrap 93% confidence intervals.

the individual income, not for family (household) income. The equilibrium value of Gini coefficient for the latter is $3/8 = 0.375$, as opposed to $1/2 = 0.5$ for the former.

An interesting evolution presents the Gini coefficient for the East-European countries (the data are supplied by [14]). During several years, G for the individual income increased from 20.42 to 24.58 in Romania, from 24.32 to 30.27 in Poland and from 23.26 to 34.2 in Hungary (Fig. 2). Even if these values are smaller then those corresponding to developed countries, the general trend of increasing is remarkable. It means an approach to the value 0.5 corresponding to the exponential distribution that seems specific to the market economies based on encounters/competition between individuals/companies/shares. Thus, the rapid growth of G is explicable for the countries in which, until the last decade of the 20th century, the totalitarian regimes had minimized the role of (fair) competition in economic development.

4 Near the equilibrium: the linear response theory

A relationship between the Boltzmann's kinetic description and Onsager's linear thermo-dynamics can be seen if we restrict attention to the kinetic equation (2) in a neighbourhood of equilibrium. Although the kinetic equation is non-linear, if we look only at small deviations around equilibrium (in the absence of an external field) we can write:

$$\rho(m,t) = \rho^0(m) + \Delta\rho(m,t) \tag{10}$$

where $\Delta\rho(x,v,t)$ is assumed to be a small change in phase space density. Substituting equation (10) into equation (2) and retaining only terms linear in $\Delta\rho$, we obtain the linearized kinetic equation:

$$\frac{\partial}{\partial t}\Delta\rho = \int \widehat{\sigma}_T\, g\rho^0\rho_1^0 \left[\frac{\Delta\rho'}{\rho'^0} + \frac{\Delta\rho'_1}{\rho'^0_1} - \frac{\Delta\rho}{\rho^0} - \frac{\Delta\rho_1}{\rho_1^0}\right] \mathrm{d}m_1 \cdot \tag{11}$$

In fact, the integral term associates a new function of x and v with the function $\Delta\rho(x,v,t)$. It is a linear functional of $\Delta\rho$ which can be written symbolically as:

$$C[\Delta\rho] = \int \widehat{\sigma}_T\, g\rho^0\rho_1^0 \left[\frac{\Delta\rho'}{\rho'^0} + \frac{\Delta\rho'_1}{\rho'^0_1} - \frac{\Delta\rho}{\rho^0} - \frac{\Delta\rho_1}{\rho_1^0}\right] \mathrm{d}m_1 \cdot \tag{12}$$

Introducing the translation operators:

$$T(m,m_1)\rho(m_1) = \rho(m) \qquad T'_1(m,m_1)\rho(m_1) = \rho(m'_1)$$

$$T'(m,m_1)\rho(m_1) = \rho(m') \qquad T_1(m,m_1)\rho(m_1) = \rho(m_1)\cdot$$

Equation (12) becomes:

$$C[\Delta\rho] = \int \widehat{\sigma}_T\, g\rho^0\rho_1^0[T' + T'_1 - T - T_1]\frac{\Delta\rho(m_1,t)}{\rho^0(m_1)}\mathrm{d}m_1$$

$$\equiv \int \Theta(m,m_1)\frac{\Delta\rho(m_1,t)}{\rho^0(m_1)}\mathrm{d}m_1 \cdot \tag{13}$$

In order to complete the Onsager's picture, we define the entropy density in phase space:

$$s = -\rho\ln\rho\cdot \tag{14}$$

The intensive variable conjugate to $N(m) = \rho\mathrm{d}m$ is:

$$F(\rho) = \frac{\partial s}{\partial\rho} = -(\ln\rho + 1)$$

and the local thermodynamic force in phase space around equilibrium is given by:

$$X = F(\rho) - F(\rho^0) = -\ln\frac{\rho}{\rho^0} \cong -\frac{\Delta\rho}{\rho^0}\cdot \tag{15}$$

Finally, introducing the operator:

$$L[X] \equiv -\int \Theta(m,m_1)X_1\mathrm{d}m_1 \tag{16}$$

the linearized kinetic equation takes the form:

$$\frac{\partial}{\partial t}(\Delta\rho) = L[X]\cdot \tag{17}$$

Equation (17), of the kind of Liouville equation, signifies at the same time the Onsager linear equation (or "Onsager regression equation" [12]) at the kinetic level of description.

Long time ago, economists have used the linear laws in studying the correlations between the macroeconomic variables. The arguments developed in this paragraph could explain, once again, from a physical point of view, both the successes and the failure of linear economic laws for long-time scales. First, the linearization is possible only in the proximity of equilibrium states; second, the kinetic level of description holds only at short-time scales (*e.g.* shorter than the time scale of economic cycles).

5 The money conservation and the time reversal symmetry

Before to test the quasi-linear laws in the proximity of equilibrium we have to remark that, in physics, the Onsager symmetry relation $L_{12} = L_{21}$ is derived from the time reversal symmetry. It is naturally to ask whether an economic system should have this property. In answer, based on some previous results [9,10], we can conclude that:

- In a closed economy, the total amount of money is conserved.
- At equilibrium, in developed countries, the money and individual income distributions are exponential, of the same kind with the Boltzmann-Gibbs distribution (8).

These empirical evidence lead to the non-trivial analogy between the amount of money and the energy of physical systems. On the other hand, let us remember that in accordance with the Noether theorem, certain conservation laws are related to general properties of space and time [15]. In particular, the energy conservation law is derived from the time reversal symmetry. Thus, we expect that this property to be also satisfied in economic equilibrium systems. However, when the methods of a certain field are applied to another field the question naturally arises whether the results correspond to reality. The only way to prove this kind of extrapolation is by analysing the empirical data series; we aims to do it in the next section.

So far we suggested that the Boltzmann gas picture could be suitable to the description between individuals or firms as the exponential distribution of money seems to be specific to market economies. Certainly, the kinetic approach is not the only suitable with this end in view: as the linear response is a general property of any system close to equilibrium, the linear laws (17) and the symmetry of kinetic coefficients can be derived from the more systematic framework of the linear response theory [16].

6 An empirical test of the linear laws in the neighbourhood of equilibrium

In accordance with Onsager linear theory [17], in the neighbourhood of equilibrium the generalized fluxes (rates of extensive parameters) are linear functions on the generalized forces (gradients of intensive parameters). An inherent difficulty in the study of macro-economic systems is related to the choice of the relevant variables for a reduced description. As we have already mentioned, such relevant (extensive) variable can be considered the total amount of money, whose (annual) rate is measured through the indicator called money supply, M. The money supply table shows money and reserve money. Money relates to the liabilities of the monetary system in currency and demand deposits to the domestic private sector. Reserve money relates to the liabilities of the monetary authorities in currency and demand deposits to deposit money banks and the domestic private sector. Money supply is measured in millions of national currency units end of period.

The choice of the intensive parameters is more difficult. We take into account that the most important classical economic schools (created by A. Walras, I. Fisher, A. Marshall, A.C. Pigou) as well as M. Friedman and Chicago economic school [18] claim that there is a close relation between the amount of money and inflation. Instead of the rate of inflation we consider the indicator called consumer index price for all items, P, which can be taken as an intensive parameter. The consumer price index numbers are designed to show changes over time in the general level of prices of goods and services that a reference population acquire, use or pay for consumption. A consumer price index is estimated as a series of summary measures of the period-to-period proportional change in the prices of a fixed set of consumer goods and services of constant quantity and characteristics, acquired, used or paid for by the reference population. Consumer price index is expressed as percentage taking a certain annual value as reference. In the database that we used P is taken as 100% for 1990. The quantity of money depends also, in the short run, on the rates of interests [11]. Because it is hard to evaluate precisely this indicator at the scale of the whole economy, we consider as generalized force only the rate of discount of the central banks, D. Rates shown represent those rates at which the central bank either discounts or makes advances against eligible commercial paper and/or government securities for commercial banks or brokers. Rates are given as percentage per annum end of period.

Table 1. Correlation coefficients between the scaled values of: money supply (M); unemployment (U); consumer price index – all items (P); rates of discount of central banks (D).

	Hungary		Switzerland		Spain	
	P	D	P	D	P	D
M	0.948	−0.983	0.816	−0.831	0.891	−0.917
U	−0.841	0.970	−0.922	0.572	−0.959	0.797

On the other hand, there is a relation between inflation and unemployment synthesized in the well-known Phillips regression curve. It claims that decreasing inflation implies linear increasing unemployment rate until a "natural" rate that corresponds to zero inflation. Although subsequent studies showed that this assumption fails at long-time scale ("the Gibson paradox"), recent works find that it is valid in the short run, validating the thesis of natural rate of unemployment [18]. Since the short-time scale is the scale at which the kinetic approach holds we choose as the second generalized flux the rate of unemployment U. The series on unemployment statistics are derived from labour force sample surveys or employment office statistics. Generally, data represent the total number economically active in thousands and of economically active persons wholly unemployed or temporarily laid off.

As a first step, we have introduced dimensionless quantities M, P, D and U scaling the variables at their initial values *i.e.* the values at the beginning of the period taken into account (the values corresponding to 1996[1] are taken equal to units – see figures legends). Thus, the Onsager linear equations near to equilibrium can be written as:

$$\begin{cases} M = L_{11}P + L_{12}D \\ U = L_{21}P + L_{22}D\cdot \end{cases} \tag{18}$$

In order to prove these relations we use the data supplied by United Nation Statistic Division [19] referring to three European countries that belong to rather different geographical areas: Hungary, Switzerland and Spain.

Table 1 includes the correlation coefficients of different series of scaled variables, calculated as:

$$C_{u,v} = \mathrm{Cov}(u,v)/\sigma_u\sigma_v$$
$$= \left[\frac{1}{n}\sum_{i=1}^{n}(u_i - \langle u\rangle)(v_i - \langle v\rangle)\right]/\sigma_u\sigma_v$$

where $u = M,\ U$; $v = P,\ D$; $\sigma_u,\ \sigma_v$ = the variances of u and v. Note that the correlation coefficient satisfies: $-1 \le C_{u,v} \le 1$. The next step was to identify pairs of values for which $D = \text{const.}$, and pairs for which $P = \text{const.}$; using these values, we obtained the phenomenological coefficients L_{ij} (Tab. 2a and b). In Figures 2a, 3a and 4a we have plotted the empirical Phillips curves $P = P(U)$. The correlation between M and D are emphasized through the curves $D = D(M)$, which we have plotted in Figures 2b, 3b and 4b.

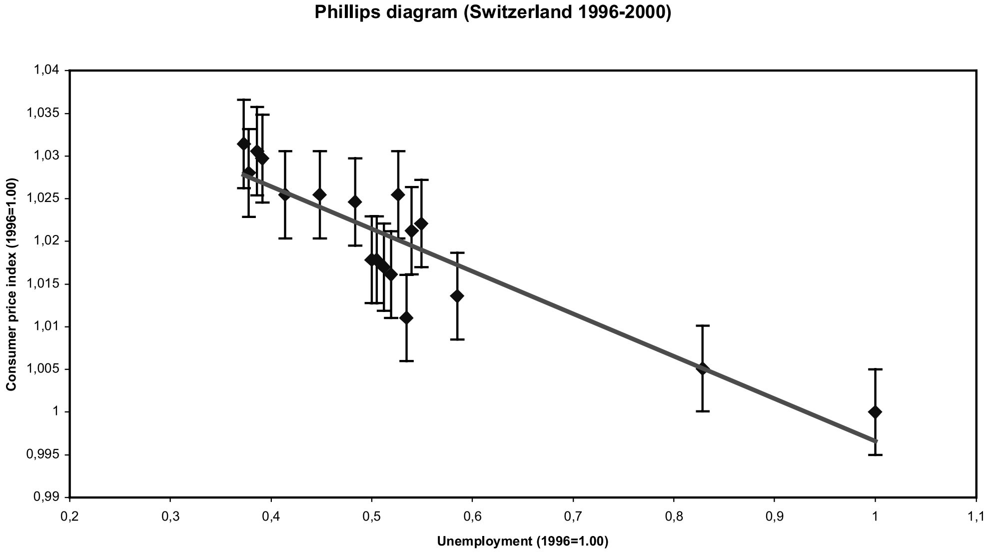
Table 2. (a) The empirical (gross) data used to calculate the phenomenological coefficients. The values marked by asterisk in two neighbouring lines are considered approximately equal. (b) The scaled data and the phenomenological coefficients: $L_{11} = (\Delta M/\Delta P)_{D=ct.}$; $L_{12} = (\Delta M/\Delta D)P_{=ct.}$; $L_{21} = (\Delta U/\Delta P)_{D=ct.}$; $L_{21} = (\Delta U/\Delta D)_{P=ct.}$. The initial data are enclosed in Table 2a. The values corresponding to 1996 (respectively 1999 for the last four lines) are taken as units (see figures legends). The errors of measurement correspond to error bars in Figures 2–4.

Country	Year/Mounth	Consumer Price Index All terms (1990=100)	Money Supply (mill. of national currency units)	Rates of Discount (percentage per annum)	Unemployement (thousands persons aged 16–64)
Hungary	2000–05	617.7	2031.650	11.00*	
	2000–06	620.8	2080.550	11.00*	
	2000–04	614.4*	1997.590	12.00	
	2000–05	617.7*	2031.650	11.00	
	2000–05	617.7		11.00*	389.6
	2000–07	627.8		11.00*	376.9
	2000–04	614.4*		12.00	411.1
	2000–05	617.7*		11.00	389.6
Switzerland	1996	117.8	128.177	1.00*	
	1997	118.3	139.776	1.00*	
	2000–08	121.1*	156.861	3.07	
	2000–10	121.5*	161.950	3.02	
	2000–10	121.5		3.02*	62.9
	2000–12	191.1		3.03*	67.7
	2000–01	120.4*		2.48	92.6
	2000–05	120.8*		2.32	62.8
Spain	2000–06	145.9	58.348	4.27*	
	2000–07	146.7	59.159	4.30*	
	2000–07	146.7*	59.159	4.30	
	2000–08	147.4*	58.235	4.41	
	2000–08	147.4		4.41*	1487.6
	2000–12	142.7		4.39*	1495.6
	1998	138.3*		3.00	1889.5
	1999	141.6*		2.72	1651.6

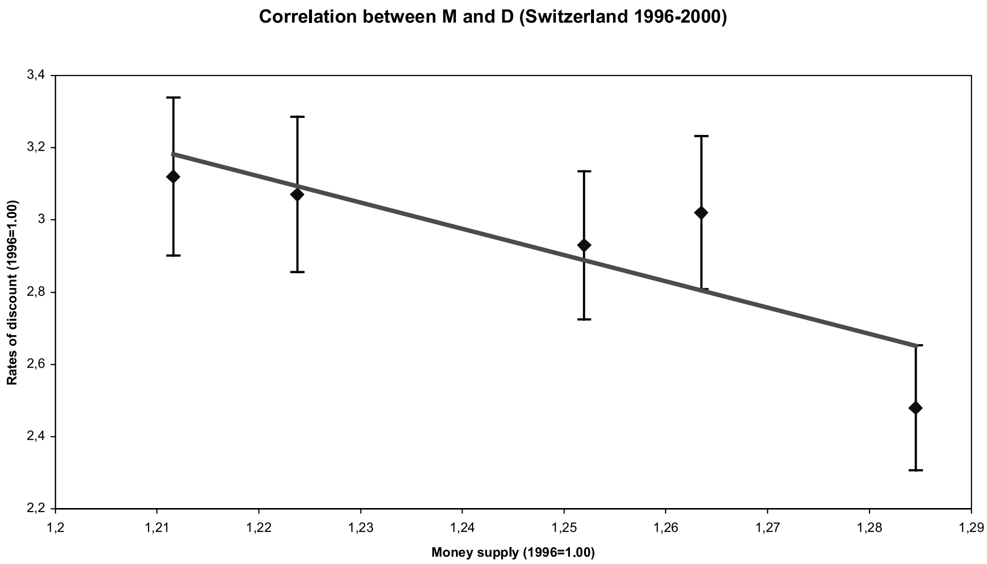
(a)

Country	P	M	D	U	L_{ij}
Hungary	1.613	1.642	0.47*		$L_{11} = 4.880 \pm 0.536$
	1.621	1.681	0.47*		
	1.605*	1.614	0.52		$L_{12} = -0.633 \pm 0.069$
	1.613*	1.642	0.47		
	1.613		0.47*	0.778	$L_{21} = -0.961 \pm 0.105$
	1.640		0.47*	0.752	
	1.605*		0.52	0.821	$L_{22} = 0.987 \pm 0.108$
	1.613*		0.47	0.778	
Switzerland	1.000	1.000	1.00*		$L_{11} = 21.319 \pm 1.492$
	1.004	1.090	1.00*		
	1.028*	1.223	3.07		$L_{12} = -0.794 \pm 0.055$
	1.031*	1.263	3.02		
	1.031		3.02*	0.373	$L_{21} = -1.397 \pm 0.097$
	1.011		3.03*	0.401	
	1.022*		2.48	0.549	$L_{22} = 0.845 \pm 0.059$
	1.025		2.32	0.413	
Spain	1.030	0.951	1.56*		$L_{11} = 2.339 \pm 0.116$
	1.036	0.964	1.58*		
	1.036*	0.964	1.58		$L_{12} = -0.372 \pm 0.018$
	1.040*	0.969	1.62		
	1.040		1.62*	0.900	$L_21 = -0.146 \pm 0.007$
	1.007		1.61*	0.905	
	0.970*		1.10	0.830	$L_22 = -0.015 \pm 0.050$
	1.000*		1.00	0.725	

(b)



(a)



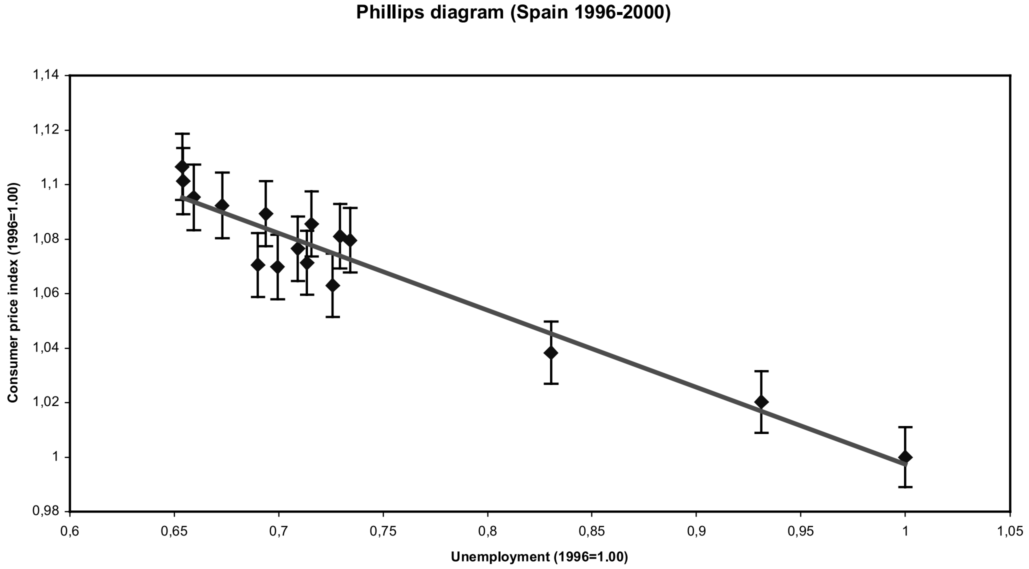
(b)

Fig. 3. (a) Consumer price index (P) *versus* unemployment (U) for Switzerland. The empirical data were registered at UNSD database [16] from 1996 to 2000. The values corresponding to 1996 ($P = 117.8\%$ for $P = 100\%$ in 1990; $U = 168.6$ thousands persons aged 16–64) are considered equal to 1.00. Error bars are bootstrap 99.5% confidence intervals. (b) Rates of discount of central banks (D) *versus* money supply (M) for Switzerland. The empirical data were registered at UNSD database [16] from 1996 to 2000. The values corresponding to 1996 ($D = 1.00\%$ per annum; $M = 128.177$ mill. francs) are considered equal to 1.00. Error bars are bootstrap 93% confidence intervals.

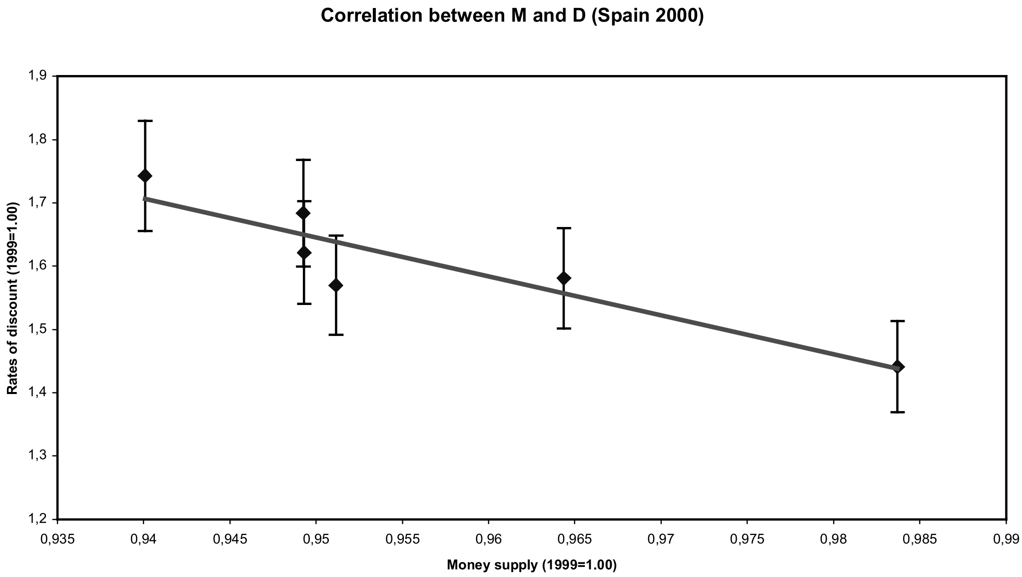
First, let us examine the results included into Table 1. A correlation coefficient greater than 0.8, as well as smaller than -0.8, cannot be the product of hazard: it involves a substratum, which emerges macroscopically as a linear (or "quasi-linear", taking into account the influence of the random noise) dependence between variables. Then, it is easy to observe that some of the basic requirements of Onsager theory [17] are fulfilled:

$$L_{11}L_{22} - L_{12}L_{21} > 0; \qquad L_{jj} > 0,\, j = 1.2\cdot \tag{19}$$

In checking the next requirement, namely the symmetry of the matrix $[\mathbf{L}]$ one has to take into account that the macroeconomic time series are too short and noisy.



(a)



(b)

Fig. 4. (a) Consumer price index (P) *versus* unemployment (U) for Spain. The empirical data were registered at UNSD database [16] from 1996 to 2000. The values corresponding to 1996 ($P = 117.8\%$ for $P = 100\%$ in 1990; $U = 168.6$ thousands persons aged 16–64) are considered equal to 1.00. Error bars are bootstrap 99.5% confidence intervals. (b) Rates of discount of central banks (D) *versus* money supply (M) for Spain. The empirical data were registered at UNSD database [16] from May 2000 to November 2000. The values corresponding to 1999 ($D = 2.72\%$ per annum; $M = 61.345$ mill. pesetas) are considered equal to 1.00. Error bars are bootstrap 95% confidence intervals.

Nevertheless, the minimum confidence interval for the linear relation between the plotted variables is 89%. The maximum relative errors correspond to Hungary (11% – Fig. 2a and 7% – Fig. 2b) while for the others series relative errors are less than 10% (0.5% – Fig. 3a; 7% – Fig. 3b; 1.1% – Fig. 4a and 5% – Fig. 4b). Unlike the correlations and linear dependences, which are well fitted by empirical data, the best estimations of the equality $L_{12} \cong L_{21}$ are affected by errors between 14% and 35%. These are related by the influence of noise on the short data series processed. Nonetheless, the accordance between the two kinetic coefficients in regard of the sign and order of magnitude allow us to conclude that the time-reversal symmetry seems to be a property of economic systems close to equilibrium as well as it actually happens for any conservative system composed by many interacting agents.

7 Concluding remarks

From long time ago, the Phillips curve has been a field of encounter between various economic schools. Today, most of them accept the quasi-linear dependence of inflation on the unemployment in the short run *i.e.* at short-time scales. Nonetheless, dependence such that plotted in Figures 2a, 3a, and 4a does not hold for any country at any time. Accordingly our working assumptions (Sect. 2), the validity of linear laws indicates that the system runs in the proximity of equilibrium. This condition as well as the supposition of short-time scales justifies the use of kinetic methods. In Sections 2–3 we found again, in an extended framework, the exponential distribution of money/income for the great majority of population. Assuming the system in a stationary state, near the equilibrium (Sect. 4), we can introduce into approach the regular increase of the amount of money (4–5% annual) required by the modern economic theories. As the total amount of money has the basic properties of energy in physical systems, the symmetry of kinetic coefficients can be related to the time reversal symmetry (Sect. 5).

The data analysed in Section 6 show that the quasi-linear laws seem not to be universal laws. Although the basically requirements of the Onsager theory are fulfilled, there are great differences between the values of phenomenological coefficients from a country to other. Moreover, it is possible that the relevant variables for a country are not relevant for another. Briefly, the choice of the relevant variables for a reduced description of macroeconomic systems rests an open question. On the other hand, the confirmation of the quasi- linear phenomenological approach near the equilibrium could open some new directions of research in econophysics: first, on the way of non-equilibrium thermodynamics (*e.g.* the significance of the production of entropy; the possible analogies with the transport phenomena), second, on the way of non-equilibrium statistics – the emergence of the bifurcation points and limit cycles. We could ask if the last are in relation with the economic cycles. The answer seems to be affirmative, taking into consideration the fail of linear laws at the end of an economic cycle. Obviously, the question requires more investigation in future.

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